November 17, 2015

Reduce the given matrix.

By rows,

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

By columns,

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -2 \\ 0 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & -2 \\ 1 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[R][A]$$

$$\left([R][A]^T\right)^T$$

$$AR^T$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 3 & 0 & 3 \\ -1 & -1 & 1 & -1 \end{bmatrix}$$

rrf

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 3 & 0 & 3 \\ -1 & -1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ -1 & -1 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ -1 & -1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ 0 & 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 3 & 0 & 3 \\ -1 & -1 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 3 & 0 & 3 \\ -1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & -1 & 0 & 3 \\ -1 & 1 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & -1 & 0 & 3 \\ -1 & 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & -1 & -2 & 3 \\ -1 & 1 & 3 & -1 \end{bmatrix}$$

1. Show that v_1, v_2, v_3 are independent but v_1, v_2, v_3, v_4 are dependent:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad v_4 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}.$$

Solve $c_1v_1 + \cdots + c_4v_4 = 0$ or $Ac = 0$. The v's go in the columns of A.

***The columns of** A **are independent exactly when** $N(A) = \{$**zero vector**$\}$.*

2. Find the largest possible number of independent vectors among

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad v_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad v_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}.$$

This number is the _____ of the space spanned by the v's.